The Trans-Planckian Problem for Inflationary Cosmology Revisited

Robert Brandenberger[1,2,3] and Xinmin Zhang[2,3]

1) Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada
2) Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918-4, Beijing 100049, P.R. China and
3) Theoretical Physics Center for Science Facilities (TPCSF), Chinese Academy of Sciences, P.R. China
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We consider an inflationary universe model in which the phase of accelerated expansion was preceded by a non-singular bounce and a period of contraction which involves a phase of deceleration. We follow fluctuations which exit the Hubble radius in the radiation-dominated contracting phase as quantum vacuum fluctuations, re-enter the Hubble radius in the deflationary period and re-cross during the phase of inflationary expansion. Evolving the fluctuations using the general relativistic linear perturbation equations, we find that they exit the Hubble radius during inflation not with a scale-invariant spectrum, but with a highly red spectrum with index $n_s = -3$. We also show that the back-reaction of fluctuations limits the time interval of deflation. Our toy model demonstrates the importance for inflationary cosmology both of the trans-Planckian problem for cosmological perturbations and of back-reaction effects. Firstly, without understanding both Planck-scale physics and the phase which preceded inflation, it is a non-trivial assumption to take the perturbations to be in their local vacuum state when they exit the Hubble radius at late times. Secondly, the back-reaction effects of fluctuations can influence the background in an important way.

I. INTRODUCTION

As was pointed out in [1] and discussed in detail in [2] (see also [3]), the usual computation of the spectrum of fluctuations in inflationary cosmology (see [4] for the original reference, [5] for a comprehensive review, and [6] for a more condensed overview) suffers from a serious conceptual problem: provided that the duration Δt of inflation exceeds $70H^{-1}$, where H is the Hubble constant during inflation, then the physical wavelengths of all scales which are being probed today through observations were smaller than the Planck length at the beginning of the period of inflation [38]. The computation of the spectrum of cosmological perturbations is based on General Relativity and classical scalar field theory, both of which are clearly not valid (and not even good approximations) on length scales smaller than the Planck length. Thus, in inflationary cosmology the fluctuations emerge from the "trans-Planckian region of ignorance" (see Figure 1).

One might have hoped that the predictions of inflation would be robust to how one models the physics of the trans-Planckian scales. However, it is easy to construct toy models of trans-Planckian physics in which large changes to the usual predictions arise [2] [39]. One can, however, impose initial state criteria for fluctuations in inflationary models which ensure that there are only minor modifications to the usual predictions of inflation, e.g. by imposing a time-like "new physics hypersurface" which lies outside of the trans-Planckian zone of ignorance, and demanding that fluctuations originate in a state which minimizes the energy at that time [7].

A further conceptual problem which scalar field-driven models of inflation do not solve is the singularity problem.

As shown in [8], a past singularity is unavoidable in scalar field-driven models of inflation.

Recently, there have been a lot of attempts to resolve the cosmological singularity by means of a cosmological bounce. Such a bounce can be achieved by considering a particularly chosen higher derivative gravity action which is free of ghosts [9, 10], and it can be obtained by introducing quintom matter [11] [40]. Ghost condensation with additional higher derivative terms may also lead to a non-singular bounce (see e.g. [13]). Bounces can also be obtained by making use of the spatial curvature term in the context of Einstein gravity [14]. The minimal length principle of string theory also points to a resolution of the singularity problem [15], possibly involving a bouncing cosmology. Finally, both brane world models [16] and loop quantum cosmology (see [17] for a recent review) can give rise to non-singular bounces (for a recent general review on bouncing cosmologies see [18]).

The first point we wish to make in this Note is that non-singular bouncing cosmologies with a post-bounce inflationary phase provide a clean framework to discuss the trans-Planckian problem of inflationary cosmology: the fluctuations can be defined at early times during the contracting phase when their physical wavelength is in the far infrared. The fluctuations then evolve in a non-singular way through the bounce into the period of inflation (for previous studies of the impact of pre-bounce evolution on the spectrum of cosmological perturbations in inflationary cosmology see [19, 20]). In a model which includes a period of deflation before the bounce, we demonstrate that, at least for a range of modes, the fluctuations will not be in a Bunch-Davies-like vacuum state on sub-Hubble scales during the phase of inflationary expansion. Thus, the trans-Planckian problem for inflationary fluctuations is revealed to be a very serious one.
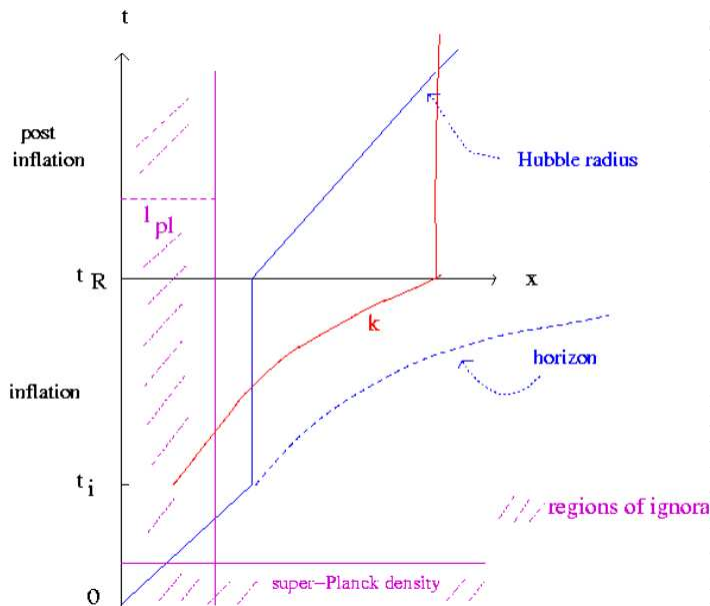
FIG. 1: Illustration of the trans-Planckian problem for fluctuations in inflationary cosmology. If the period of inflation is sufficiently long, then at the beginning of the inflationary phase the physical wavelength of all scales k which are being observed today is smaller than the Planck length l_{pl}. In the plot, the vertical axis is time, and the horizontal axis denotes physical distance. The period of inflation lasts from t_i to the time of reheating t_R. The dashed regions indicate zones where one cannot trust the effective equations of motion derived from General Relativity. These break down on length scales smaller than the Planck length, and at energy densities which are sufficiently close to the Planck density m_{pl}^4.

The second point which we make in this Note is that a pre-bounce phase of exponential contraction is unstable to the back-reaction effects of fluctuations which enter the Hubble radius during the contraction. Due to the fact that fluctuations grow on super-Hubble scales, they rather generically obtain a red spectrum, and lead to a back-reaction energy-momentum tensor which dominates over the vacuum contribution leading to deflation. This leads to a constraint on the range of modes for which large trans-Planckian effects are predicted (see e.g. [21, 22, 23, 24] for work pointing out that demanding that inflationary expansion is robust puts constraints on the

amplitude of trans-Planckian effects).

The outline of this note is as follows. In the following section we review the trans-Planckian problem for inflationary cosmology and give a brief overview of previous approaches. In Section 3 we present a particular bouncing cosmology with a phase of inflationary expansion. We compute the spectrum of fluctuations starting with a Bunch-Davies vacuum state in the early stages of the contraction. We find that the squeezing of the fluctuations on a super-Hubble scales during the period of contraction leads to a spectrum of fluctuations which is not close to scale-invariant.

II. THE TRANS-PLANCKIAN PROBLEM FOR COSMOLOGICAL PERTURBATIONS

The initial discussions of the trans-Planckian problem for fluctuations were based on considering non-trivial dispersion relations for modes at frequencies larger than the Planck length [2]. It was found that suitable choices of dispersion relations can lead to large trans-Planckian effects, including order unity changes in the spectral index of cosmological perturbations (see e.g. [25]). What is happening is that modes evolve non-adiabatically on length scales smaller than the Planck length. The time interval which modes spend at sub-Planck length scales depends on the wave-number, and hence short wavelength modes are more excited than long wavelength modes. The magnitude of the trans-Planckian effects is constrained by demanding that the energy in short wavelength modes does not prevent inflation [21, 22, 23]. In this case, interesting trans-Planckian effects with amplitude of the order H/m_{pl} for scalar metric fluctuations can be found.

If one assumes that modes exit the Hubble radius in the local vacuum state [26], then the trans-Planckian effects are very small (namely of order $(H/m_{pl})^2$. However, the whole point of the trans-Planckian problem for fluctuations is that there is no reason to expect that modes should be in the local vacuum state when they exit the Hubble radius.

In another approach to the trans-Planckian problem, a "new physics hypersurface" can be introduced corresponding to the Planck length, and it can then be assumed that fluctuations emerge in the local vacuum state [7]. In this case, the amplitude of the trans-Planckian effects is bounded from above by H/m_{pl}.

Imposing initial conditions for all modes at a fixed time breaks the translation symmetry which ensures the scale-invariance of the spectrum. But such effects are red-shifted as the duration of the inflationary phase is increased (see e.g. [27]) if the calculations are done in the context of the usual inflationary effective field theory.

Other approaches to the trans-Planckian problem include choosing different initial state prescriptions (the so-called α vacua [28]), considering the effects of space-

space [29] or space-time [30] uncertainty relations on the evolution of fluctuations, and studying the effects of ultraviolet cutoffs [31]. For a more comprehensive review of different approaches to the trans-Planckian problem see [25].

As we study in this Note, bouncing cosmologies with a phase of inflationary expansion offer another setup to study the trans-Planckian problem for inflationary fluctuations. This issue was initially considered in [19], where it was found that the trans-Planckian problem in models with a modified dispersion relation persists in a bouncing cosmology. In [20], a bouncing inflationary cosmology was studied in which there is no phase of deflation before the bounce. In this case, interesting signatures of the pre-bounce physics in the spectrum of cosmological fluctuations were found, but these effects were small in amplitude.

In this Note, we consider a bouncing inflationary cosmology which contains a period of deflation before the bounce. In this case, a range of scales of interest crosses the Hubble radius four times - first they exit the Hubble radius during the pre-deflation phase, then they enter during the deflationary phase, they exit again during inflation, to finally re-enter at late times. For these scales, we will find very large trans-Planckian effects. In fact, it is the large effect of these modes which limits the duration of the deflationary phase.

III.   FLUCTUATIONS IN A SPECIFIC BOUNCING INFLATIONARY COSMOLOGY

A.   Background

In this Note, we will be considering a bouncing inflationary cosmology which contains a deflationary period before the bounce. The specific nature of the physics providing the non-singular bounce will not be relevant for our analysis. To be specific, we may assume that the bounce is provided by the specific higher derivative gravity model of [9, 10].

We will model both the radiation phase and the inflationary phase with a scalar field φ with a potential

$$V(\varphi) = \frac{\lambda}{4}\varphi^4 . \tag{1}$$

For field values $|\varphi| \ll m_{pl}$ (where m_{pl} is the Planck mass) the field is oscillating and the time-averaged equation of state is that of radiation (see e.g. [32]), whereas for field values $|\varphi| \gg m_{pl}$ the field is slowly rolling and hence leads to an equation of state $w \sim -1$ which yields inflation [41]. We will first describe the background dynamics in the absence of back-reaction of the fluctuations. In the following section we will then discuss how the back-reaction modifies the background dynamics.

We begin the evolution in a contracting radiation phase in which φ is oscillating with a small amplitude. Due to Hubble anti-friction during the contracting phase, the

field amplitude grows, eventually leading to a transition to a deflationary phase with $w \sim -1$. This transition takes place at a time which we denote by $-t_2$. Once the energy density reaches the critical value where the physics that determines the bounce sets in, the universe will undergo a non-singular bounce, after which a period of inflationary expansion will set in during which φ is slowly rolling down its potential. Once $|\varphi| \sim m_{pl}$, a smooth transition to a radiation phase will set in. A space-time sketch of our scenario is given in Figure 2.
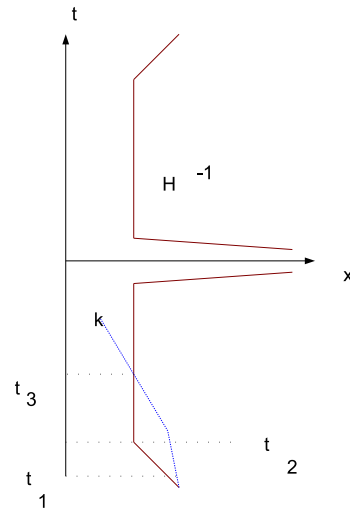

FIG. 2: Space-time sketch of our bouncing inflationary cosmology (in the absence of the back-reaction of fluctuations). Time is along the vertical axis, physical length along the horizontal axis. The red curve denotes the Hubble radius $H^{-1}(t)$, the blue curve denotes the physical wavelength corresponding to a fixed co-moving scale k. This scale exits the Hubble radius during the phase of radiation-dominated contraction at the time $-t_1(k)$ (the minus sign and the k-dependence are not included in the figure), the transition to the deflationary phase occurs at the time $-t_2$, and the scale re-enters the Hubble radius during the deflationary phase at the time $-t_3(k)$ (once again, the minus sign and the k-dependence are not included in the sketch).

As can be seen from Fig. 2, fluctuations originate on sub-Hubble scales during the radiation-dominated contracting phase. They exit the Hubble radius at time $-t_1(k)$ during this phase. We are interested in scales which re-enter the Hubble radius during the contracting deflationary phase (at a time which we denote by $-t_3(k)$) and exit the Hubble radius once again during the phase of inflationary expansion, before finally re-entering the Hubble radius at late times (at the time $t_1(k)$).

B. Fluctuations: Formalism

In the context of inflationary cosmology it has proven to be convenient [33, 34, 35] to track the cosmological fluctuations in terms of the variable ζ, the curvature fluctuations in co-moving coordinates. This variable is conserved at phase transitions and is constant on super-Hubble scales in an expanding universe. The variable ζ is related to the metric fluctuation $\Phi(\mathbf{x}, \eta)$ in longitudinal gauge (in which the metric takes the form [42]

$$ds^2 = a^2(\eta)\big[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2\big], \quad (2)$$

where η is conformal time and the $\mathbf{x}$ denote co-moving spatial coordinates) via

$$\zeta = \frac{2}{3}\big(\mathcal{H}\Phi^{'} + \Phi\big)\frac{1}{1+w} + \Phi, \quad (3)$$

$\mathcal{H}$ denoting the Hubble expansion rate in conformal time, a prime indicating the derivative with respect of η, and $w = p/\rho$ being the equation of state parameter (p and ρ are pressure and energy density, respectively).

The variable ζ is closely related to the variable v [5] in terms of which the action for cosmological fluctuations has canonical kinetic term:

$$\zeta = \frac{v}{z} \quad (4)$$

where for hydrodynamical matter

$$z(\eta) = \frac{1}{c_s\theta} \quad (5)$$

where c_s is the speed of sound and

$$\theta = \frac{\mathcal{H}}{a}\Big[\frac{2}{3}(\mathcal{H}^2 - \mathcal{H}^{'})\Big]^{-1/2}. \quad (6)$$

The equation of motion for the Fourier mode v_k of v is

$$v_k^{''} + \Big(k^2 - \frac{z^{''}}{z}\Big)v_k = 0. \quad (7)$$

This shows that on length scales larger than the Hubble radius, where the k^2 term is negligible, v is frozen in (i.e. it does not oscillate), whereas on sub-Hubble scales v_k is oscillating with approximately constant amplitude.

The equation of motion for v has a singularity at the bounce point which is associated with the fact that at this point the comoving gauge is not well-defined. This issue was discussed for example in Refs. [36]. However, the problems are due to singularities in the factor $\frac{z^{''}}{z}$. Since we are interested in scales which are sub-Hubble near the bounce, this problem does not arise for us. Since the equation of motion for ζ is easier to solve during the transition from radiation-domination to deflation, we will follow ζ.

C. Fluctuations in the Contracting Radiation Phase

On super-Hubble scales, the equation of motion for v_k in a universe which is contracting or expanding as a power p of time, i.e.

$$a(t) \sim t^p, \quad (8)$$

is given by

$$v_k^{''} = \frac{p(2p-1)}{(p-1)^2}\eta^{-2}v_k, \quad (9)$$

which has solutions

$$v(\eta) \sim \eta^\alpha \quad (10)$$

with

$$\alpha = \frac{1}{2} \pm \nu, \quad \nu = \frac{1}{2}\frac{1-3p}{1-p}. \quad (11)$$

In the radiation phase we have $p = 1/2$ and hence $\nu = -1/2$ which leads to the two values for α which are $\alpha = 1$ and $\alpha = 0$. Hence,

$$v_k(\eta) = c_1\eta + c_2, \quad (12)$$

where c_1 and c_2 are constants, and hence (making use of $a(\eta) \sim \eta$)

$$\zeta_k(\eta) = c_1 + c_2\eta^{-1}. \quad (13)$$

In a contracting universe, the second term dominates, and we conclude that ζ is growing on super-Hubble scales.

D. Fluctuations in the Contracting Deflationary Phase

During deflation we have

$$a(\eta) \sim \eta^{-1}, \quad (14)$$

with η tending to infinity as the bounce is approached. From (11) we see that the two solutions of the equation for v have values

$$\alpha = 2 \quad \text{and} \quad \alpha = -1. \quad (15)$$

Hence,

$$\zeta_k(\eta) = c_3\eta^3 + c_4, \quad (16)$$

where c_3 and c_4 are constants. The first mode is the growing mode

Now we want to calculate the spectrum of fluctuations when they enter the Hubble radius during the deflationary phase, starting with vacuum initial conditions in the early contracting radiation phase, i.e. with the spectrum

$$\zeta(k, -t_1(k)) = \frac{v}{a}(k, -t_1(k)) \sim \frac{k^{-1/2}}{a(-t_1(k))} \sim k^{1/2}, \quad (17)$$

making use of the Hubble radius crossing condition in the radiation phase

$$a(-t_1(k))k^{-1} = 2t_1(k) \qquad (18)$$

which leads to

$$\eta(t_1(k)) \sim k^{-1} . \qquad (19)$$

Next, we compute the spectrum at the end of the radiaton phase, making use of (13)

$$\zeta(k, -t_2) = \frac{\eta(-t_1(k))}{\eta_2}\zeta(k, -t_1(k)) \sim k^{-1/2} , \qquad (20)$$

where we have made use again of (19).

Finally, we compute the spectrum at Hubble radius re-entry in the deflationary phase:

$$\zeta(k, -t_3(k)) = \big(\frac{\eta(-t_3(k))}{\eta(t_2)}\big)^3 \zeta(k, -t_2) . \qquad (21)$$

Making use of the fact that in the deflationary phase the Hubble crossing condition gives

$$\eta(-t_3(k)) \sim k^{-1} \qquad (22)$$

we immediately obtain

$$\zeta(k, -t_3(k)) \sim k^{-7/2} . \qquad (23)$$

Our result (23) shows that the initial vacuum spectrum has been transformed into a extremely red spectrum. The redness of the spectrum is a consequence of the fact that ζ is growing on super-Hubble scales in the contracting phase, and that long wavelength modes are super-Hubble for a much longer time than short wavelength modes.

E. Fluctuations during and after the Bounce

During the bounce, the variable v_k is oscillating, hence ζ will have the same amplitude at Hubble radius crossing during inflation after the bounce as it had at Hubble radius crossing before the bounce. After Hubble radius crossing in the expanding phase, ζ is conserved. Hence at late tiimes t

$$\zeta(k, t) = \zeta(k, t_3(k)) \simeq \zeta(k, -t_3(k)) \sim k^{-7/2} , \qquad (24)$$

which corresponds to a power spectrum

$$P_\zeta(k) \equiv \frac{k^3}{12\pi^2}|\zeta(k)|^2 \sim k^{-4} , \qquad (25)$$

i.e. spectral index $n_s = -3$. This is a manifestation of the severity of the trans-Planckian problem for fluctuations in inflationary cosmology. Without new physics affecting the evolution of fluctuations, we have a model in which fluctuations emerging from inflation are highly non-scale-invariant.

IV. BACK-REACTION EFFECTS

As emphasized in [21, 22] and explored in more detail in [23], the magnitude of trans-Planckian effects which grow towards the UV end of the spectrum is bounded by its back-reaction on the background. In the following we will see that back-reaction effects tightly constrain the length of a deflationary contracting phase if the modes which are entering the Hubble radius during the deflationary period are not in their (Bunch-Davies) vacuum state.

We can estimate the energy density ρ_f in sub-Hubble scale perturbations in the following way. First of all, recall (see e.g. [5, 6]) that for scalar field matter

$$v = a\big(\delta\varphi + \frac{z}{a}\Phi\big). \qquad (26)$$

Since on sub-Hubble scales the energy density is dominated by matter and since the matter fields are oscillating, the energy density can be well approximated by the gradient energy of $\delta\varphi$. Since on sub-Hubble scales we can approximate $\delta\varphi = a^{-1}v$, and making use of the fact that v_k oscillates with constant amplitude, we have

$$\rho_f(t) \simeq a^{-2}(t) \int_{k_{\min}(t)}^{k_{\max}} d^3\mathbf{k} a^{-2}(t)k^2 v(k, \eta_3(k))^2 , \qquad (27)$$

where $k_{\max}$ corresponds to the wavelength which crosses the Hubble radius at time $-t_2$ and $k_{\min}$ corresponds to the mode which is entering the Hubble radius at time t.

Making use of the scaling of v in the radiation phase and in the deflationary phase we find

$$v(k, \eta_3(k)) = v(k, \eta_2)\big(\frac{\eta_3(k)}{\eta_2}\big)^2 = v(k, \eta_1(k))\big(\frac{\eta_3(k)}{\eta_2}\big)^2 \qquad (28)$$

and hence

$$\begin{aligned} \rho_f(t) &\simeq a^{-4}(t) \int_{k_{\min}(t)}^{k_{\max}} d^3\mathbf{k} k^2 |v(k, \eta_1(k))|^2 \big(\frac{\eta_3(k)}{\eta_2}\big)^4 \\ &\sim a^{-4}(t) \int_{k_{\min}(t)}^{k_{\max}} dk k^4 k^{-1} \big(\frac{k_{\max}}{k}\big)^4 \\ &\sim a^{-4}(t) k_{\max}^4 \ln\big(\frac{k_{\max}}{k_{\min}}\big) . \end{aligned} \qquad (29)$$

Since $a^{-1}(t_2)k_{\max} = H$, we find

$$\rho_f(t) \simeq \big(\frac{a(t_2)}{a(t)}\big)^4 H^4 \ln\big(\frac{k_{\max}}{k_{\min}}\big), \qquad (30)$$

which is smaller than the background energy density provided that the period of deflation is short.

As expected, the energy density of these sub-Hubble modes scales as radiation. Hence, as soon as

$$\big(\frac{a(t_2)}{a(t)}\big) > \big(\frac{m_{pl}}{H}\big)^{1/2} \qquad (31)$$

(up to logarithmic factors) then the radiation in the fluctuation modes will begin to dominate over the constant energy density driving deflation, and a transition to another radiative contraction phase will set in. Hence, the number $\tilde{N}$ of e-foldings of the deflationary phase is bounded by

$$\tilde{N} < \frac{1}{2}\log(\frac{m_{pl}}{H})\,, \qquad (32)$$

where m_{pl} denotes the Planck mass. Thus, if we want to have a sufficiently long period of inflation in the expanding phase, enough for inflation to solve the problems of Standard Big Bang cosmology such as the horizon and flatness problems [37], then we cannot have a symmetric bounce.

Another consequence of this result is that the range of wavelengths which exhibit the $n = -3$ spectrum is bounded. If N denotes the number of e-foldings of inflation in the expanding phase, then only modes which exit the Hubble radius during the inflationary phase more than $N - \tilde{N}$ e-foldings before the end of inflation exhibit the modified spectrum. Modes which exit the Hubble radius later are in the usual Bunch-Davies since they were never super-Hubble during the contracting phase.

We conclude that if the inflationary phase lasted a large number of e-foldings, then the trans-Planckian effects from the contracting phase are red-shifted to wavelengths which are still super-Hubble today. This result is analogous to that obtained in [27] who show that initial condition signatures which carry deviations from the usual scale-invariance of the spectrum of cosmological perturbations are red-shifted to the far infrared as N increases.

However, in the context of a bouncing cosmology, we are not free to simply choose initial conditions for the scalar field φ driving inflation such that a very large number of e-foldings results. If we start, as assumed here, with the scalar field oscillating in the far past in the contracting phase, then it is very unlikely that a large value of φ will be generated after the bounce. Hence, we expect that N will be close to the minimal value required for inflation to solve the problems of Standard Big Bang cosmology, and hence the trans-Planckian signatures discussed in this paper will be on observable scales

V. CONCLUSIONS AND DISCUSSION

We have presented a model which demonstrates the severity of the trans-Planckian problem for cosmological fluctuations in inflationary cosmology. Embedding a period of inflationary expansion into a non-singular model with a cosmological bounce which contains a deflationary phase of contraction, we have shown that vacuum initial conditions for fluctuations in the early stages of the contracting universe leads to a spectrum of perturbations which emerge from the period of inflationary expansion which is not scale-invariant. In the example in which the initial phase of contraction is dominated by radiation, we have shown that a power spectrum with spectral index $n_s = -3$ results.

Our analysis does not make use of any non-standard evolution of the fluctuations - these are traced from the time when they first exit the Hubble radius in the contracting phase to the present time using the general relativistic fluctuation equations.

By the nature of the trans-Planckian problem for cosmological fluctuations, we cannot trust the validity of the fluctuation equations on length scales smaller than the Hubble scale. What we have shown, however, is that since the fluctuations dip into the trans-Planckian region during the contracting deflationary phase with a spectrum which is very different from the vacuum spectrum, it would require un-natural fine-tuning of trans-Planckian physics to convert the fluctuations into a spectrum which looks vacuum-like during the expanding inflationary phase.

We have seen that back-reaction effects of fluctuations tightly constrain the duration of the deflationary phase before the bounce. This constrains the range of fluctuation modes which are subject to the trans-Planckian corrections discussed here. If the deflationary phase lasts $\tilde{N}$ e-foldings, and the inflationary phase of expansion N e-foldings, then only modes which exit the Hubble radius during the expanding period more than $N - \tilde{N}$ e-foldings before the end of inflation exhibit the modified spectrum. However, in the context of our setup, it is highly improbable to get a long period of inflation. Hence, it is very likely that the trans-Planckian signatures will be in the observable range of wavelengths.

In conclusion, we have presented an inflationary model with a preceding cosmological bounce in which cosmological fluctuations emerge with a $n_s = -3$ spectrum and not with a scale-invariant spectrum. This demonstrates that inflationary cosmology must deal with the trans-Planckian problem for fluctuations. Conversely, our results also imply that if our universe in fact underwent a period of inflation, then Planck-scale physics can be probed with current cosmological observations.

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[38] The numerical coefficient 70 depends mildly on the scale of inflation: we have chosen the scale to be that of Grand Unification, the scale which is required for simple inflaton potentials.

[39] By large we mean order unity.

[40] A particular realization of the quintom bounce can be obtained by considering the scalar field sector of the Lee-Wick model [12].

[41] We use the standard notation where the equation of state parameter w is the ratio $w = \frac{p}{\rho}$, with p and ρ being pressure and energy density, respectively.

[42] We are neglecting anisotropic stress.